Pergamon

Ocean Engineering 26 (1999) 83–98

OCEAN ENGINEERING

Technical Note

Estimation of wave directional spreading in shallow water

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Received 4 April 1997; accepted 18 June 1997

Abstract

This paper describes wave directional spreading in shallow water. Waves were measured for a period of 2 months using the Datawell directional waverider buoy at 15 m water depth on the east coast of India in the Bay of Bengal. The study also showed that in shallow water wave directional spreading was narrowest at peak frequency and widened towards lower and higher frequencies. The wind direction was found to deviate from the wave direction during most of the time. The unidirectional spectrum was found to be satisfactorily represented by Scott spectra. © 1998 Elsevier Science Ltd. All rights reserved.

Keywords: Directional spectra; Spreading function; Wave direction

1. Introduction

Knowledge of the distribution or spreading of wave energy around the propagating direction is of importance for a wide range of engineering and scientific applications (Wiegel and Mobarek, 1966; Goda et al., 1978; Deo and Narasimhan, 1988; Chaplin

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PII: S0029-8018(97)10014-2

et al., 1993). This includes estimation of wave loads on offshore structures, long-term estimation of waves and estimation of sediment transport.

According to the principle of superposition of linear waves, the sea state is composed of a large number of individual wave components, each having a particular wave number (k), frequency (f) and direction of propagation (θ) (Sarpakaya and Isaacson, 1981). Most conveniently a two-dimensional spectrum $S(f,\theta)$ is generally adopted to describe the real sea shape in continuum of frequency and direction regardless of wave numbers. Physically $S(f,\theta)dfd\theta$ represents contribution to the variance due to waves with frequencies between f and $f + df$ and direction between θ and $\theta + d\theta$, so that total variance of the sea surface elevation is given by

$$\sigma_\eta^2 = \int_0^\infty \int_{-\pi}^{\pi} S(f,\theta)df\, d\theta \tag{1}$$

One of the common methods to obtain the two-dimensional energy spectrum, $S(f,\theta)$ is through its expression as the product of the one-dimensional energy spectrum $S(f)$ and its angular distribution function $D(f,\theta)$ at each frequency (Borgman, 1969).

$$S(f,\theta) = S(f)\cdot D(f,\theta) \tag{2}$$

where

$$\int_{-\pi}^{\pi} D(f,\theta)d\theta = 1 \tag{3}$$

Many models are available to represent the wave directional spreading (Pierson et al., 1955; Cote et al., 1960; Longuet-Higgins et al., 1963; Donelan et al., 1985). Due to simplicity and general effectiveness in representing the wave directional spreading, the cosine power model, originally proposed by Longuet-Higgins et al. (1963) is widely used. This is given as below

$$D(f,\theta) = G(S)\cos^{2s}[(\theta - \theta_m)/2] \tag{4}$$

where

$$G(S) = \frac{2^{2s}}{2\pi}\frac{\Gamma^2(s + 1)}{\Gamma(2s + 1)} \tag{5}$$

and Γ is the gamma function, s is the spreading parameter and θ_m is the mean wave direction. Here the value of the parameter s controls directional spreading around the mean wave direction. Cartwright (1963) showed that s can be related to the Fourier coefficients a_0, a_1, b_1, a_2 and b_2 in two ways, s_1 and s_2, where,

$$s_1 = \frac{r_1}{1 - r_1} \text{ or } s_2 = \frac{1 + 3r_2 + (1 + 14r_2 + r_2^2)^{1/2}}{2(1 - r_2)} \tag{6}$$

where

$$r_1 = \frac{\sqrt{a_1^2 + b_1^2}}{a_0} \text{ and } r_2 = \frac{\sqrt{a_2^2 + b_2^2}}{a_0} \tag{7}$$

For fairly steady wind condition and mild sea states parameter s can be expressed as a function of relative frequency (f/f_m) and wave age (c_m/U) (Mitsuyasu et al., 1975), as under:

$$s_3 = 11.5(c_m/U)^{2.5}(f/f_m)^b \tag{8}$$

where

$$b = 5 \text{ for } (f/f_m) \leq 1$$

$$= -2.5 \text{ otherwise.}$$

In the above equations c_m is wave speed associated with peak frequency (f_m) and U is measured wind speed at 10 m height above mean sea level.

Wang (1992) showed that s can be related to design wave height and wave period as under:

$$s_4 = 0.2(H_s/L_p)^{-1.28}\,(f/f_m)^b \tag{9}$$

where L_p is wave length associated with peak frequency of the spectrum (f_m) estimated on the basis of the linear dispersion relation and H_s is significant wave height. This allows the design engineer to use design wave height and wave period without relating to the wind condition for estimating the value of s.

For the design of coastal structures and floating breakwaters the directional wave spectrum at shallow water is needed. The spreading parameter s estimates the directional spreading around the wave travelling direction. The objective of the present study is to estimate the parameter s from measured shallow water wave data and represent it by empirical equations.

2. Data measurement and analysis

Wave measurement was carried out using the Datawell directional waverider buoy, which is spherical in shape with a diameter of 0.9 m. It is equipped with a heave–pitch–roll sensor (Hippy-40), a three axis fluxgate compass, two fixed accelerometers and a microprocessor. The roll–pitch information from the Hippy sensor is used to convert x and y accelerations in the moving buoy reference plane to acceleration along fixed, horizontal north–west axes. All three accelerations are then digitally integrated to displacements and filtered (Stephen and Tor Kollstad, 1991). The buoy was deployed at 15 m water depth on the east coast of India in the Bay of Bengal. Sea bed contours at that region are straight and parallel to the coast. The waves will be generally high along this coast during the northeast monsoon period (October–

January). Wave data were therefore collected during November and December 1995. The sampling interval was 0.78125 s and data was recorded for 20 min duration at every 3 h interval. The data analysis is carried out by using the analysis technique proposed by Kuik et al. (1988). In this method characteristic parameters of $D(f,\theta)$ at each frequency are obtained directly from Fourier coefficients a_0, a_1, b_1, a_2 and b_2 without any assumption of model. Fourier coefficients are estimated from auto, co- and quadrature spectra of the collected buoy signals.

The directional parameters obtained are:

$$\text{directional width, } \sigma = \sqrt{2(1-m_1)} \tag{10}$$

$$\text{skewness, } \gamma = \frac{-n_2}{[2(1-m_1)]^{3/2}} \tag{11}$$

$$\text{kurtosis, } \delta = \frac{6-8m_1+2m_2}{[2(1-m_1)]^2} \tag{12}$$

where m_1, m_2 and n_2 are centered Fourier coefficients and can be estimated from Fourier coefficients, a_0, a_1, b_1, a_2 and b_2.

The two representative wave directions defined by Longuet-Higgins et al. (1963) as follows are also estimated.

$$\text{Mean wave direction, } \theta_m = \arctan(b_1/a_1) \tag{13}$$

$$\text{Principal wave direction, } \theta_p = 0.5\arctan(b_2/a_2) \tag{14}$$

Directional width is an index of directional spreading while skewness and kurtosis give information about the shape of directional spreading function i.e., they indicate whether directional distribution is unimodal or symmetric. To get an estimation of directional spreading Goda et al. (1981) defined another parameter viz., long crestedness parameter (τ) along with the mean spreading angle (θ_k) as follows.

$$\tau = \left[\frac{1-\sqrt{a_2^2+b_2^2}}{1+\sqrt{a_2^2+b_2^2}}\right]^{1/2} \tag{15}$$

$$\theta_k = \arctan\left[\frac{\sqrt{0.5b_1^2(1+a_2)-a_1b_1b_2+0.5a_1^2(1-a_2)}}{a_1^2+b_1^2}\right] \tag{16}$$

The long crestedness parameter varied from 0 to 1 and the mean spreading angle varied from 0 to $\pi/2$. Both these parameters will be zero for unidirectional wave.

During the period of wave data collection an Aanderaa self-recording weather station was installed on the open coast at 10 m height above the mean sea level and the wind speed and direction were recorded at every 3 h interval.

3. Results and discussion

During the study period significant wave height (H_s) varied from 0.5 to 2 m while the wind speed varied from 0.5 to 7 m/s. This is shown in Fig. 1. For the same period Fig. 2 indicates the joint distribution of significant wave height and zero crossing wave period (Tz). It shows that Tz varied from 3 to 9 s. The maximum wave height (H_{max}) determined from each 20 min record based on the zero-crossing method shows that it is 1.5 times the significant wave height (Fig. 3). Following the Rayleigh distribution for an average wave period of 4–5 s, the maximum wave height has to be 1.6 to 1.7 times the significant wave height. The difference may be due to the assumption of linear, Gaussian, narrow band free surface and of independent consecutive waves in shallow water, made in the Rayleigh distribution application.

A comparison of mean wave direction and principal wave direction is carried out and shown in Fig. 4(a and b). It shows that mean wave direction and principal wave direction for frequencies ranging from 0.07 to 0.25 Hz (close to peak frequency) is the same (Fig. 4(a)). But mean wave direction and principal wave direction corresponding to higher (>0.25 Hz) and lower ($<$ 0.07 Hz) frequencies are not the same (Fig. 4(b)). Also principal wave direction is obtained only in the range 0 to π, whereas mean wave direction is obtained in the range 0 to 2π (see Eq. (13) and Eq. (14)). So mean wave direction only is considered in further studies and the same can be considered as the most representative index.

The mean wave direction corresponding to peak frequency varied from 30 to 120° with respect to north (Fig. 5). The orientation of the coastline at this region is north–south and is a straight and long open coast. The wind direction recorded at the coast shows that it varied predominantly between 0 and 90° and between 270 and 360° (Fig. 5). This shows that waves are generated in deep water and that transformed wave characteristics are only recorded by the buoy, since the waverider buoy was only 5 km away from the coast in 15 m water depth. This indicates that care has to be taken while assuming mean wave direction to be the same as the wind direction in the modelling of shallow water waves. Donelan et al. (1985) also showed that waves with frequencies near spectral peak can travel at off-wind angles in a non-stationary or fetch-limited condition. They noted that these angles spread up to 50° in Lake Ontario.

It was found that directional width, σ (Eq. (10)) was marginally more than the mean spreading angle, θ_k (Eq. (16)) (Fig. 6). The difference between these parameters is not much, with a correlation coefficient of 0.985. Goda et al. (1981); Benoit (1992); Besnard and Benoit (1994) observed similar values for these parameters.

The Fourier coefficient related parameters s_1 and s_2 (Eq. (6)) are compared with each other in Fig. 7(a). Here the values of s_1 is found to be smaller than s_2, which is consistent with the observations of Cartwright (1963); Mitsuyasu et al. (1975); Hasselmann et al. (1980); Tucker (1987); Wang (1992). The difference may be due to bimodality of the spectra, noise in the system (Tucker, 1989) and limitation on resolution in case of buoy data. The effect of noise on s_1 and s_2 by numerical simulation was studied by Ewing and Laing (1987) who reached the conclusion that s_1 is more sensitive than s_2 to noise and hence recommended the use of s_2. In the

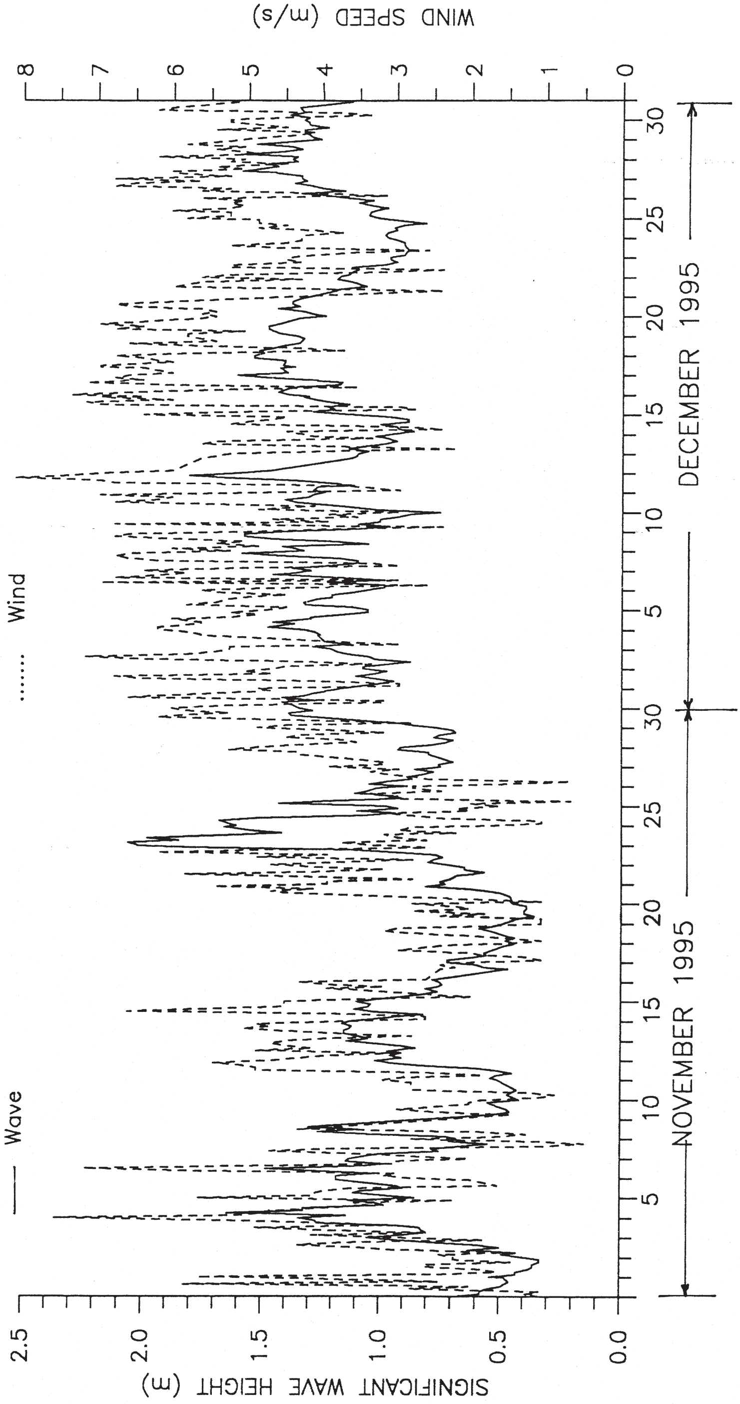

Fig. 1. Variation of significant wave height and wind speed.

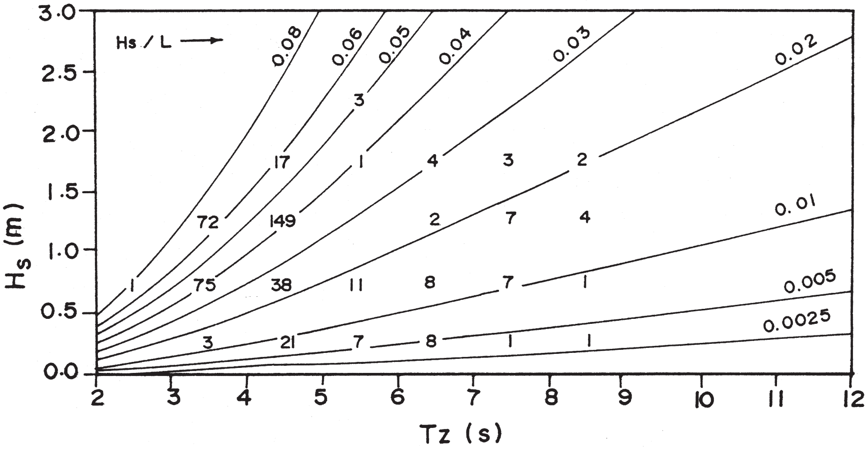

Fig. 2. Joint distribution of significant wave height and mean wave period.

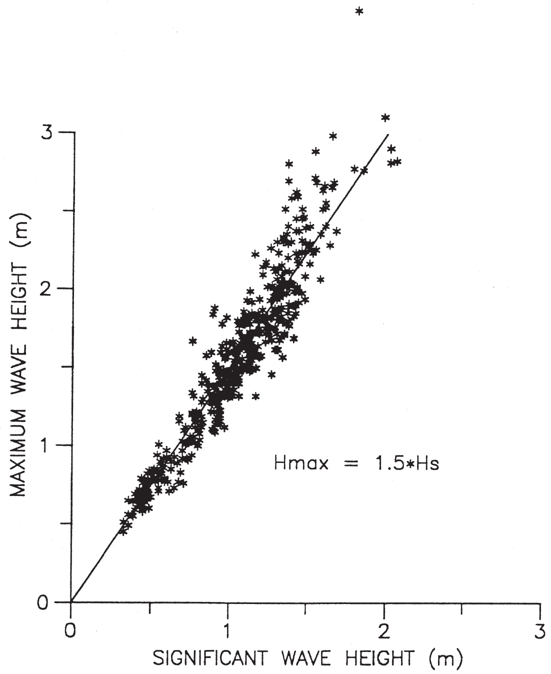

Fig. 3. Variation of maximum wave height with respect to significant wave height.

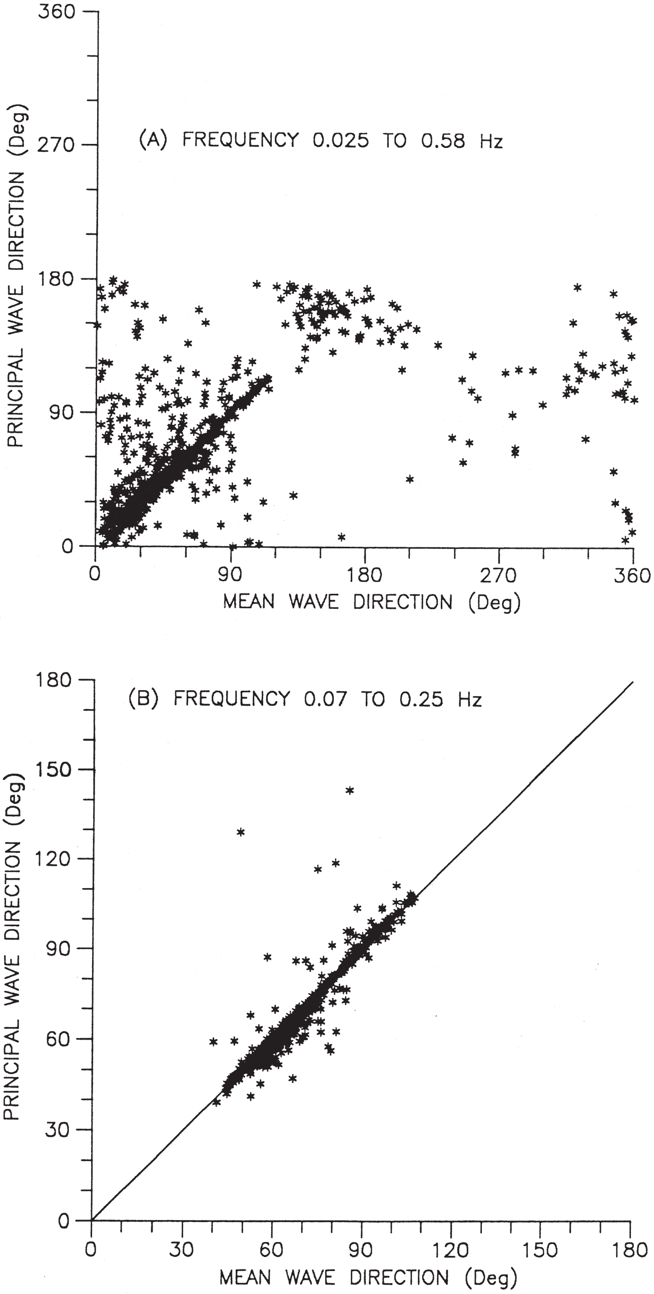

Fig. 4. Correlation between principal wave direction and mean wave direction.

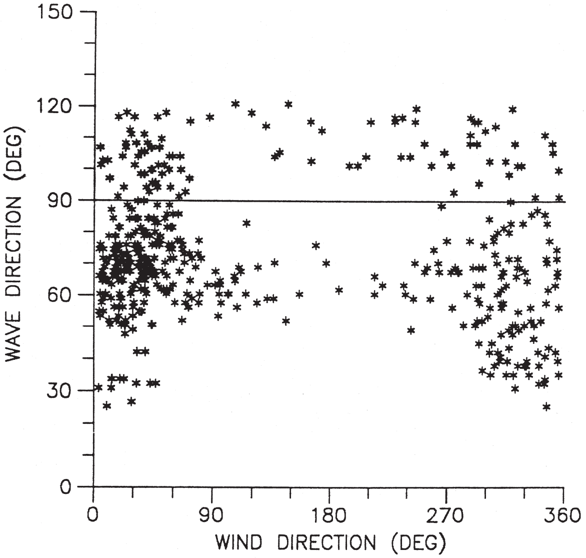

Fig. 5. Correlation between wave direction and wind direction.

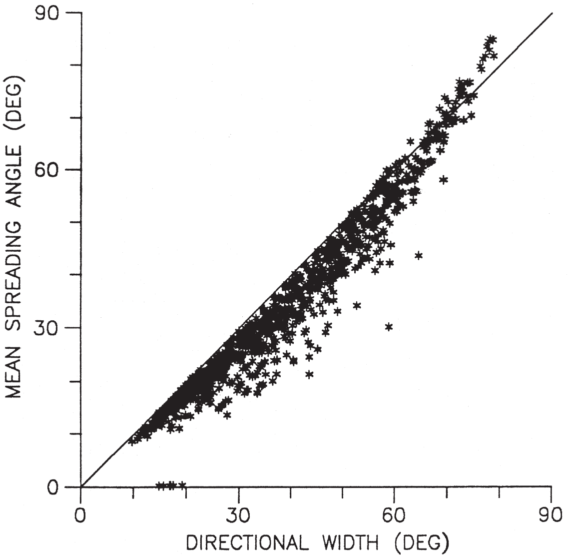

Fig. 6. Correlation between mean spreading angle and directional width.

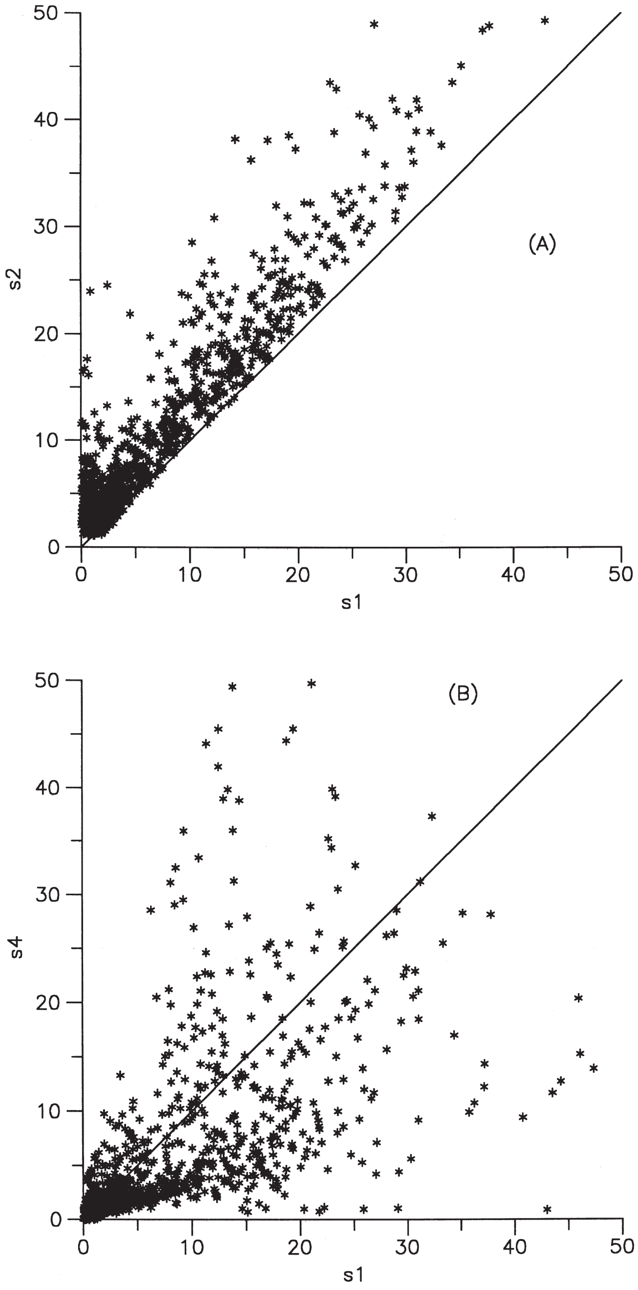

Fig. 7. Correlation between spreading parameters.

present study the difference between s_1 and s_2 was not found to be as large. A better estimate of spreading parameter s in such a situation would be the average of s_1 and s_2 (Cartwright, 1963). The spreading parameters s_3 obtained from Eq. (8) were found not to be reliable for this location since the waves are not generated locally and the difference in wave direction and wind direction is large. Hence this parameter is not considered. The spreading parameter s_4 estimated based on significant wave height and wave period (Eq. (9)) was compared with the spreading parameter s_1 and presented in Fig. 7(b). The empirical Eq. (9) for estimating s_4 is modified by changing the coefficient from 0.2 to 0.14. It is found that s_4 is more than s_1 but the difference between s_4 and s_1 is not large. Since the expression for s_4 involves only significant wave height and the period corresponds to maximum energy, it can be used in situations when only the design parameters are specified.

The variation of wave spectrum, recorded on 25 November 1995 at 09.00 h with frequency is shown in Fig. 8(a). Fig. 8(b) shows the variation of mean wave direction, principal wave direction and wind direction with frequency. It is found that the mean wave direction and principal wave direction differ at higher and lower frequencies. The local wind direction was consistent with the wave direction at higher frequencies. This indicates that the waves at higher frequencies are generated by local wind. The variation of directional width and mean spreading angle with frequency is shown in Fig. 8(c). These two parameters show similar values. The variation of s_1, s_2 and s_4 with frequencies is presented in Fig. 8(d). It is found that s_4 values are marginally higher than s_1 values. A two-dimensional spectral model can easily be represented by considering the product of a unidirectional spectrum and a directional spreading model. A lot of research has been carried out in modelling the unidirectional spectrum. For different locations and climatic conditions a number of different semi-empirical spectral models are proposed (Sarpakaya and Isaacson, 1981). For the west coast of India, the Scott spectrum (Scott, 1965) was recommended for use by Dattatri et al. (1977); Narasimhan and Deo (1979); Sanil Kumar et al. (1994) by analysing the data collected off Mangalore, Bombay High and Karwar, respectively. The probable reason might be the fact that the validation of Scotts spectrum was carried out using considerable swell dominated data. A similar situation usually prevails along many sites along the Indian coast. The Scott spectrum is given by

$$
\begin{aligned}
s(\omega) &= 0.214 H_s^2 \exp\left\{ - \left[\frac{(\omega - \omega_p)^2}{0.065(\omega - \omega_p + 0.26)} \right]^{1/2} \right\} \\
&\quad - 0.26 < (\omega - \omega_p) < 1.65 \qquad (17) \\
&\quad = 0 \text{ otherwise}
\end{aligned}
$$

where H_s is the significant wave height and ω_p is the peak angular wave frequency.

The spectrum computed from measured data along with the Scott spectrum is presented in Fig. 8(a). It can be seen that the Scott spectrum approximates the observations in a fairly satisfactory way.

The main disadvantage in using the cosine power model is that it can not model the bimodality of the sea states. Cartwright (1963); Borgman and Yfantis (1978) suggested use of the cosine power model for storm waves assuming that they are

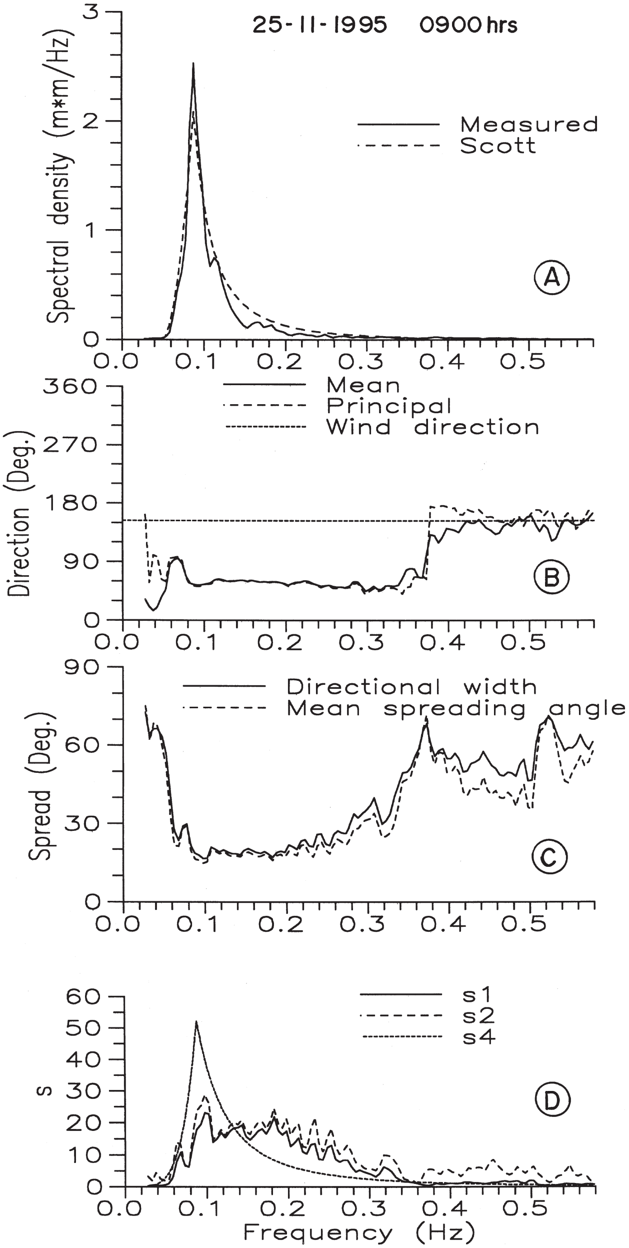

Fig. 8. Variation of spectral energy, wave direction, spread and spreading parameter with frequency.

directionally unimodal. Borgman and Yfantis (1979); Lawson and Long (1983) showed the occurrence of directional bimodality within a frequency band. Near the coast the second direction may be due to the reflection of some of the swell energy from the coast or by nearby islands (Pawka, 1983). It is important to identify whether the directional distribution is unimodal or bimodal before representing it by suitable parametric models. In the present study the following three criteria were considered to examine whether the directional spreading is unimodal or bimodal.

1. Based on the mean spreading angle, θ_k, (Eq. (16)) and long crestedness parameter, τ, (Eq. (15)) Kobune et al. (1985) proposed this criteria. Using a graph of θ_k and τ the unimodal cases and the bimodal cases are located on separate portion of the graph bounded by a curve (Fig. 9(a)).
2. Based on skewness, γ, (Eq. (11)) and kurtosis, δ, (Eq. (12)), Kuik et al. (1988) proposed this criteria. A combination of skewness and Kurtosis was plotted and they define the domain where the unimodal and symmetric directional spreading should lie (Fig. 9(b)).
3. Besnard and Benoit (1994) introduced this criteria based on the following two parameters:

$$r = \frac{\sqrt{a_2^2 + b_2^2}}{\sqrt{a_1^2 + b_1^2}} \text{ and } \Delta\theta = \min\{|\theta_m - \theta_p|, |\theta_m - \theta_p - \pi|\} \qquad (18)$$

Bimodal cases are characterised by high values of both r and $\Delta\theta$, as shown by the curve in Fig. 9(c), while unimodal cases are characterised by low values of either r or $\Delta\theta$.

Based on the studies it is found that at shallow water the criteria based on mean spreading angle and long crestedness width parameter and the criteria based on r and $\Delta\theta$ shows more reliable results for differentiating unimodal and bimodal sea state.

4. Conclusions

It was found that the wind direction deviated from the wave direction recorded during the same period, which was due to the fact that waves were recorded in nearshore, where waves were almost parallel to the coast.

The mean wave direction was found to be a more representative index than principal wave direction.

The mean spreading angle and the directional width showed similar variation with directional spread marginally more than the mean spreading angle.

The observed relatively large values of the spreading parameter near the peak frequency to lower and higher frequencies also agreed fairly well with suggestions in the literature for shallow water waves.

It was found that the values of s_2 were always more than s_1 and the average value

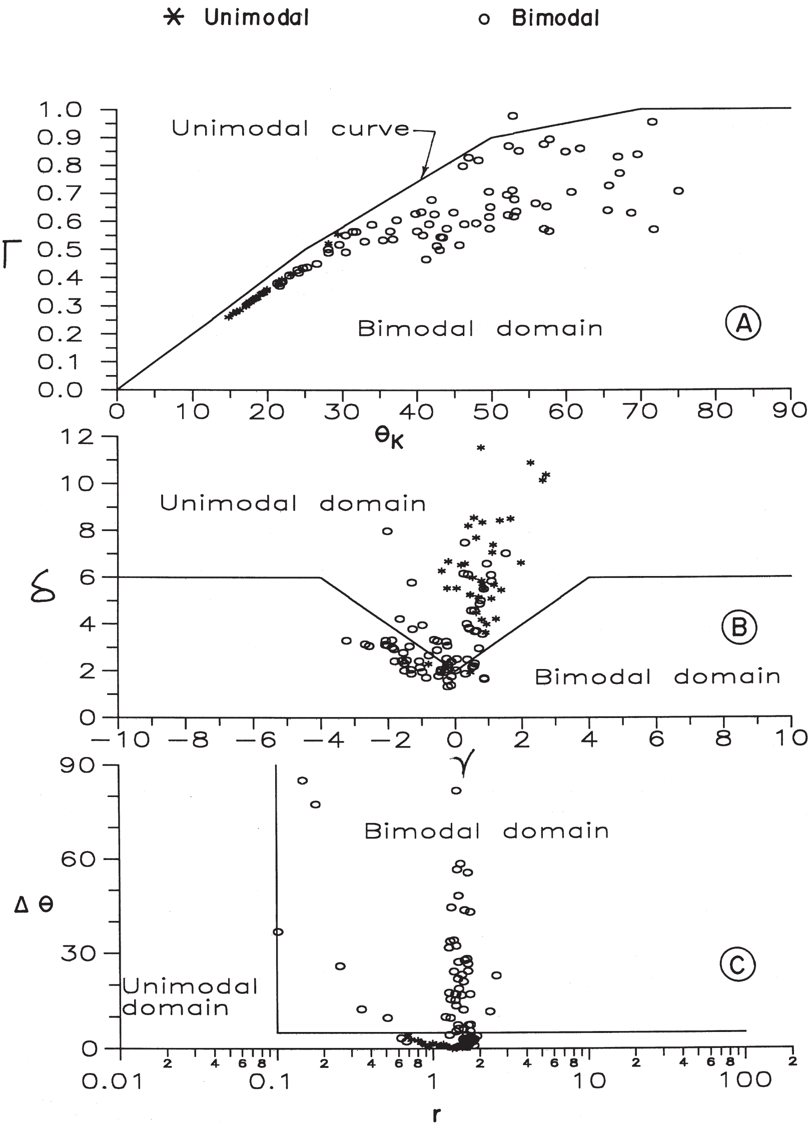

Fig. 9. Criteria for unimodal and bimodal condition.

can be used for modelling the directional spectra, but for design purposes when only the design wave parameters are specified, spreading parameter can be estimated from Eq. (9).

Unidirectional spectra can be modelled using the Scott spectra.

Acknowledgements

The authors would like to thank the Department of Science and Technology, New Delhi, India, for funding the project “Directional Wave Modelling”, the Director, National Institute of Oceanography, Goa, for the encouragement and support and staff of the Ocean Engineering Division, National Institute of Oceanography, Goa, who were involved in the data collection programme. NIO Contribution number 2569.

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