Numerical Modelling of Nearshore Wave Transformation

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ABSTRACT

A software has been developed for numerical refraction study based on finite amplitude wave theories. Wave attenuation due to shoaling, bottom friction, bottom percolation and viscous dissipation has also been incorporated. The software was successfully used to study the wave convergence causing erosion along the Campal beach in Goa. It is user-interactive in FORTRAN 77 and can find various applications.

INTRODUCTION

When deep water waves propagate into shallow water, they undergo changes in wave height, length, celerity and direction of propagation due to shoaling, refraction, bottom friction, bottom percolation, viscous dissipa tion, etc. Coastal engineers are generally supplied with wave characteristics at deep water or at some intermediate depth. The longshore sediment transport rate is to be estimated based on the longshore wave energy flux at breaking. Hence, it is important to accurately estimate wave transformation to evaluate the breaking wave characteristics. In the present study, a software has been developed for computing the wave attenuation due to shoaling, refraction, bottom friction, bottom percolation and viscous dissipation using finite amplitude wave theories.

WAVE THEORY

Wave phenomenon is complex and difficult to describe mathematically. The wave theories put forward by Airy [1] and Stokes [18] predict the wave motion reasonably well in the region where the water depth is large compared to the wavelength. The higher order wave theories [4,18] are found satisfactory under certain circumstances in describing the waves. For shallow water regions, cnoidal wave theory [9] is generally used to predict the wave form and associated motion. In very shallow regions, solitary wave theory [2,7,8,11,15] can be used to describe the wave behaviour satisfactorily. The regions of validity of various wave theories have been indicated by Le Mehaute [10].

Using the third order equations, Miche [12] has given the following relationship for wave celerity:

$$c=(gT/2\pi)\tanh kh(1+\pi H/L)^2 K \qquad \ldots (1)$$

where $K=(5+2\cosh 2kh+2\cosh^2 2kh)/(8\sinh^4 kh)$; H = wave height; L = wavelength; g = acceleration due to gravity; T = wave period; and h water depth.

The cnoidal solution for wave celerity, c has been given by Svendsen [19]:

$$c=(gh(1+A(m))H/h)^{0.5} \qquad \ldots (2)$$

where

$$A(m)=(2/m)-1-(3/m)(E/K).$$

The wave celerity in solitary wave has been given by Svendsen [19]:

$$c=(gh(1+H/h))^{0.5} \qquad \ldots (3)$$

Wave transformation

The assumptions made in estimating the nearshore wave transformation are: i) Waves are long crested and of constant period, ii) curvature of the wave front is small, so that it has a negligible effect on the velocity of propagation, iii) effects of wind, current and reflection from beaches are negligible, iv) changes in bottom topography are gradual, and v) there is no crest breaking during propagation.

Figure 1 shows the volume of water enclosed over the full depth between the two adjacent orthogonals and two vertical sections perpendicular to these orthogonals. Assuming that no energy propagates across the orthogonals:

$$b_2 E_{f2}=b_1 E_{f1}-\Delta E_{1,2} \qquad \ldots (4)$$

where b_1 = distance between orthogonals at section (1); b_2 = distance between orthogonals at section (2); E_{f1} = energy flux at section 1; E_{f2} = energy flux at section 2; and $\Delta E_{1,2}$ = loss of energy between sections 1 and 2.

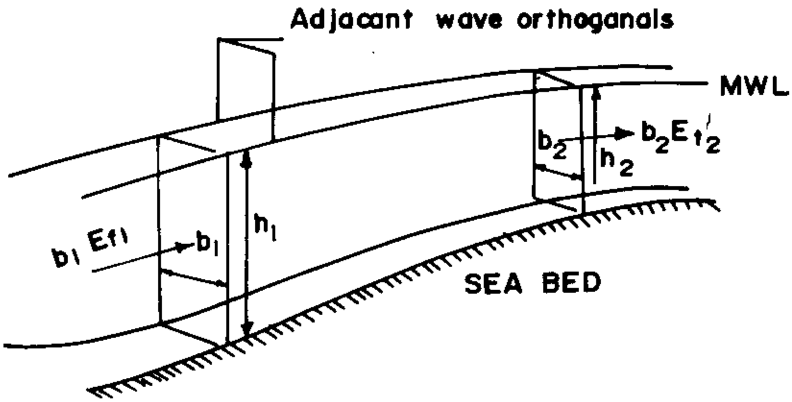

Fig. 1—Energy flux between sections through adjacent wave orthogonals

Shoaling

Assuming that the waves are approaching parallel to the straight and parallel bottom contours and hence no refraction is encountered, we have $b_1 = b_2$. Assuming the equation for wave energy flux wave energy flux (E_f) [20]:

$$K_s = (C_0/2nc)^{0.5} \quad \ldots (5)$$

where $n = 0.5(1 + (2kh/\sinh 2kh))$ and K_s = shoaling coefficient.

Refraction

Considering section 1 at deep water ($h_1 > 0.5\ L_0$) and section 2 at desired depth h:

$$H/H_0 = K_s(b_0/b)^{0.5} = K_s K_r \quad \ldots (6)$$

where K_r, the refraction coefficient $= (b_0/b)^{0.5}$. For given bottom topography and deepwater wave characteristics, the refracted orthogonals can be plotted by geometrical procedure [16] or numerical methods [5,6,13,14,17].

Considering that the dissipative forces are due to bottom friction, bottom percolation and viscous dissipation, the mean energy flux between the orthogonals O_1 and O_2 in Fig. 2 is given by:

$$E_f D_f = (\rho g H_*^2/16)\ c(1+G)\ D_f \quad \ldots (7)$$

where D_f is the elementary length of wave front between the orthogonals. Taking the ratio between Eq. (7) and the energy flux at deep-water arbitrary starting point:

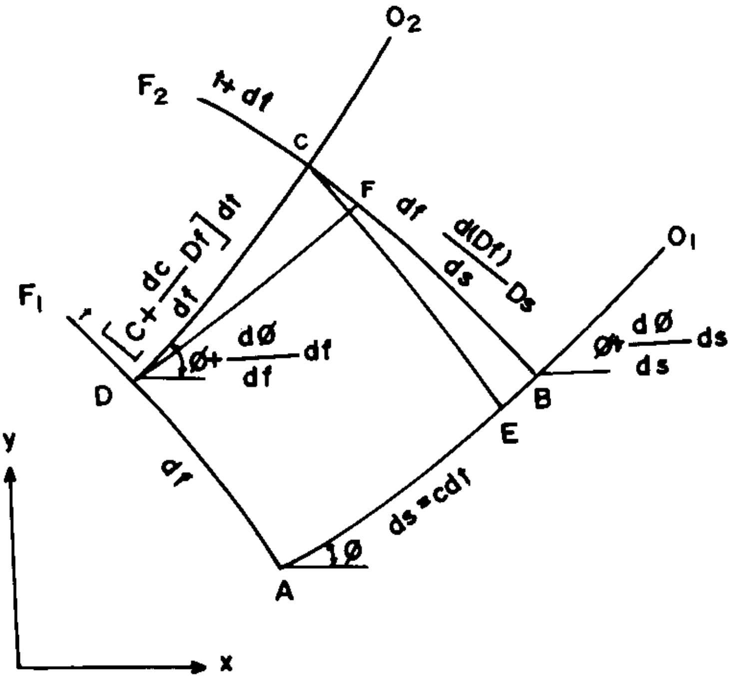

Fig. 2—System of wave orthogonals and fronts

$$H/H_0 = K_s\, K_r K_{dis} \qquad \ldots (8)$$

where

$$K_{dis} = K_f\, K_p\, K_v \qquad \ldots (9)$$

K_s, K_r, K_f, K_p, K_v are the shoaling, refraction, bottom fraction, bottom percolation and viscous dissipation constants.

Numerical wave refraction

Based on the results of Skovgaard *et al.*[17] Fig. 2 shows two adjacent orthogonals O_1, O_2 and two consecutive wave fronts F_1, F_2 separated by time interval dt. At point A, the infinitesimal distances between orthogonals and fronts are D_f and D_s respectively, where

$$D_s = c\,dt \qquad \ldots (10)$$

The distance s is taken as positive in the direction of wave propagation and the positive direction of f is such that D_s and D_f form a right hand coordinate system. θ is the angle from the X-axis to the orthogonal, positive in anti-clockwise direction.

The basic equations for the wave orthogonal become:

$$dx/dt = c\cos\theta \qquad \ldots (11)$$

$$dy/dt = c\sin\theta \qquad \ldots (12)$$

$$d\theta/dt = (\partial c/\partial x)\sin\theta - (\partial c/\partial y)\cos\theta \qquad \ldots (13)$$

For the calculation of wave heights along the orthogonal, Munk and Arthur [13] have derived a second order homogeneous ordinary differential equation for the orthogonal separation factor (β), which can be written in time t as:

$$(d^2\beta/dt^2) + p(t)(d\beta/dt) + q(t)\beta \doteq 0 \qquad \ldots (14)$$

where

$$p(t) = -2\left(\cos\theta\frac{\partial c}{\partial x} - \sin\theta\frac{\partial c}{\partial y}\right) \qquad \ldots (15)$$

$$q(t) = c[(\sin^2\theta\,\partial^2 c/\partial x^2) - \sin^2\theta\,\partial^2 c/\partial c\,\partial y) + (\cos^2\theta\,\partial^2 c/\partial y^2)] \qquad \ldots (16)$$

$$\beta = D_f/D_f st = K_r^{-2} \qquad \ldots (17)$$

Eqs (11), (12), (13) and (14) with Eqs (15) and (16) can be solved numerically with proper initial boundary conditions.

Bottom friction, bottom percolation and viscous dissipation

The nonlinear first order differential equation [19] for bottom friction is:

$$dK_f/dt = -\left(\frac{8}{3L}\frac{dc}{dh}\right) am\,fe\,K_f \qquad \ldots (18)$$

where

$$\frac{dc}{dh} = \frac{c}{h}\frac{G}{1+G} \qquad \ldots (19)$$

The nonlinear first order differential equation [19] for bottom percolation is:

$$\frac{dK_p}{dt} = -KD\frac{1}{C_0}\frac{dc}{dh}k_p \qquad \ldots (20)$$

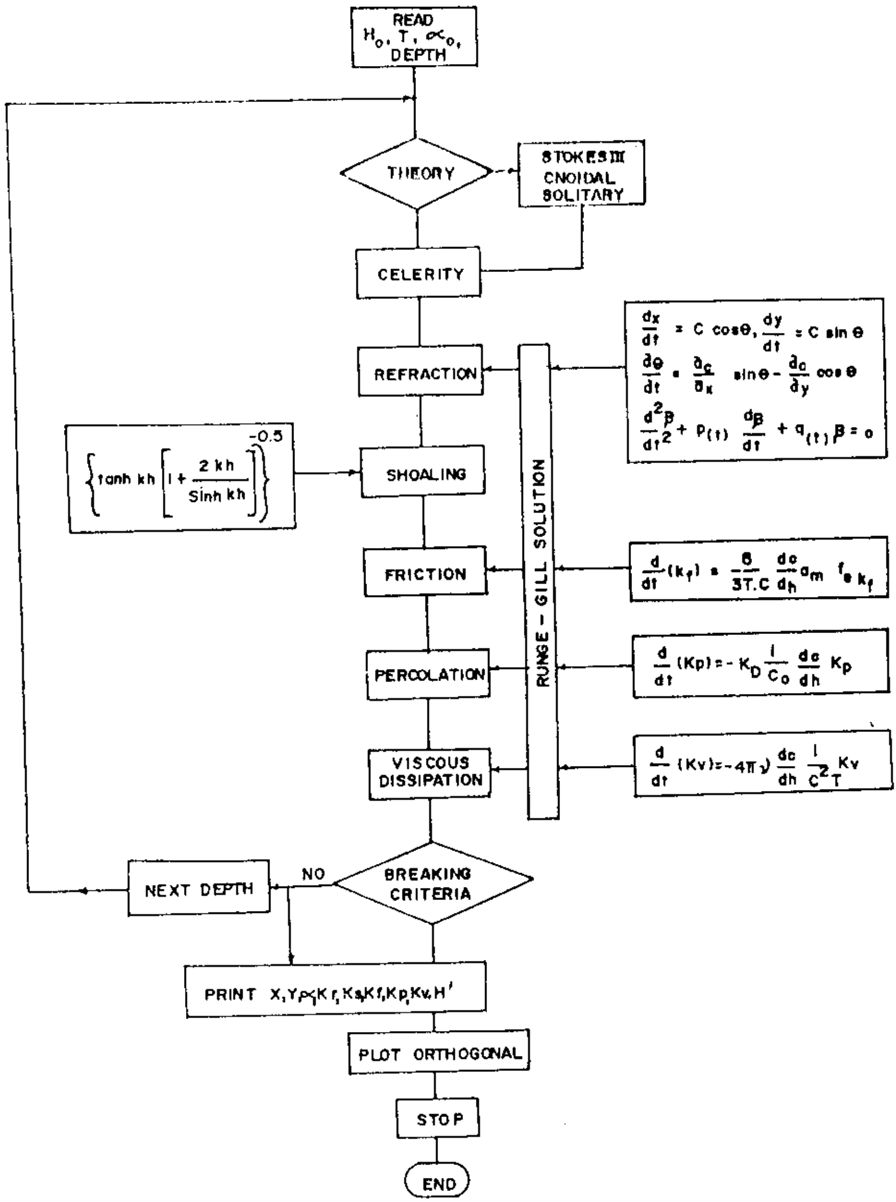

Fig. 3—Nearshore wave transformation model TRANGAM

The nonlinear first order differential equation [19] for viscous dissipation is:

$$\frac{dK_v}{dt} = -4\pi\nu \frac{dc}{dh} \frac{\sin h 2kh}{c^2 T} K_v \qquad \ldots (21)$$

Formulation of nearshore wave transformation model

Numerical model TARANGAM has been developed for computing the wave alteration due to bottom friction, percolation and viscous dissipation in addition to shoaling and refraction effects. The model has been applied to Goa coast and the results are discussed. The flow chart of the model TRANGAM is shown in Fig. 3. For a given deep water wave characteristics (H_0, T, θ) and nearshore bathymetry, it computes the wave characteristics at breaking or at any intermediate depth [3].

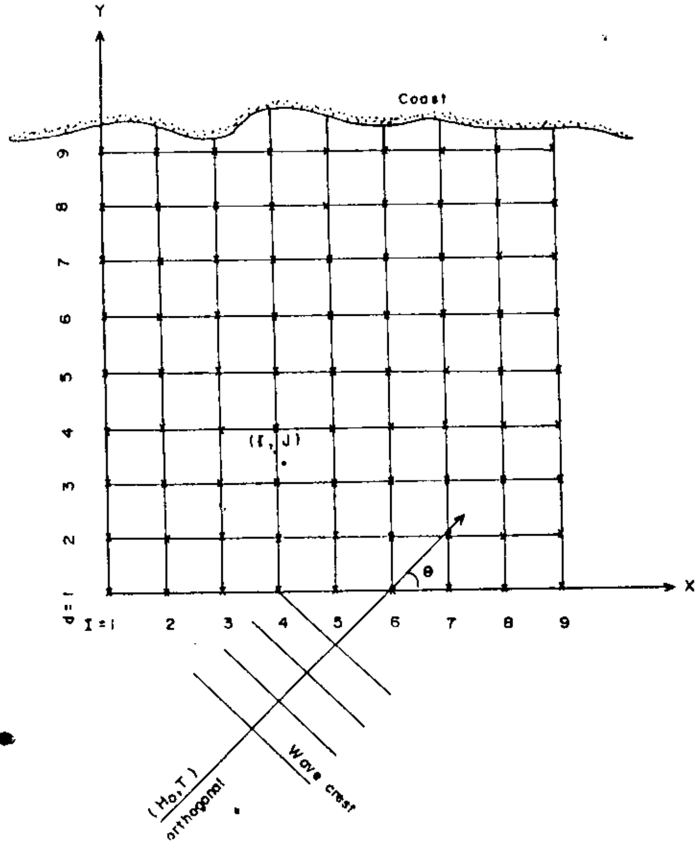

Fig. 4—System of grids

In using the finite amplitude wave theory, the selection of appropriate wave theories for the different regions is classified according to the relative water depth as shown below:

h/L_0	Wave theory
>0.2	Stokes III order
$0.2>=h/L_0>0.05$	Cnoidal
$0.05>=h/L_0>hb/L_0$	Solitary

Eqs (1) to (3) were used for computing the wave celerity according to different regions of wave theory. Eq. (5) was used for computing wave shoaling. The differential equations [Eqs (11) to (16)] were used for the numerical estimation of wave refraction. The grid system is shown in Fig. 4, where X-axis is parallel to the coastline, Y-axis is perpendicular to the coastline and θ is the angle of the wave orthogonal with the X-axis. The differential equations [Eqs (19)to (21)] are solved for obtaining bottom fric-

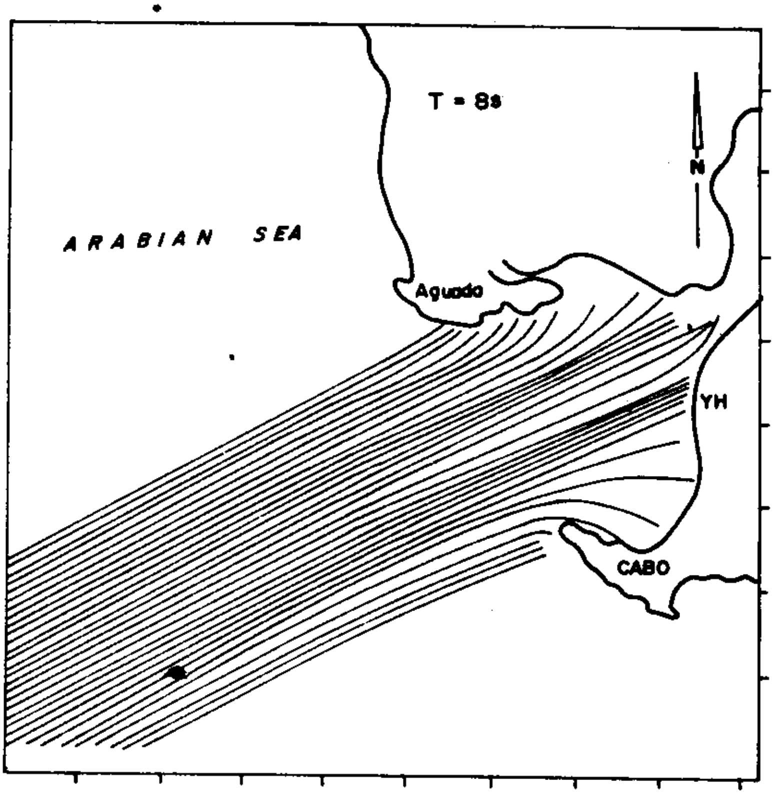

Fig. 5—Wave refraction—Goa coast

tion, bottom percolation and viscous dissipation. The various inputs for the model are given in Appendix-1.

Verification of the model

Figure 5 shows the part of Goa coast in Aguada Bay, where erosion is reported during southwest monsoon near Youth Hostel and part of Caranzalem beach. The present model is used to identify the wave refraction and the wave energy distribution along this coast. This nearshore bathymetry is taken from the hydrographic survey carried out at the National Institute of Oceanography, Goa. The inputs are the predominant wave direction WSW, $H_0 = 2m$, $T = 8s$. The output shows convergence of wave orthogonals near Youth Hostel and part of the Caranzalem beach, confirming actual field observations, indicating concentration of wave energy leading to severe erosion of the shoreline.

CONCLUSION

The present study indicates that the use of finite amplitude wave theory in the nearshore region is found to be appropriate. In addition to bottom friction, the effects of bottom percolation and viscous dissipation are also considered in the calculation of wave height dissipation. The percolation loss may be considerable at certain segments of the Indian coast, where the seabed consists of gravel for which $KD > 0.0005$ m/s. Viscous dissipation is considerable, especially along the Kerala coast, where the phenomenon of mud bank occurs during monsoon. The numerical model TARANGAM can be used for estimating nearshore wave characteristics from known deep water wave parameters.

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APPENDIX—1

Figure 4 shows the grid system adopted and accordingly the following inputs have to be arranged:

1. Total number of grids in X direction = IXEND
2. Total number of grids in Y direction = JYEND
3. Contour depths at nodal points = D(IXEND, JYEND)
4. Distance between the grids (metres) = SCA
5. Slope of the seabed = SLOPE
6. Starting point of wave orthogonal in X direction = X
7. Starting point of wave orthogonal in Y direction = Y
8. Deep water wave height (m) = H
9. Wave period (s) = T
10. Direction of wave crest with X axis (deg) = THE
    (wave crest to X axis anti-clockwise positive)
11. Wave theory = HIGHER
12. Time step = T
13. Stop computation at = BREAKING/GIVEN DEPTH
14. Nikuradse roughness parameter = AKN
15. Permeability coefficient = AKD
16. Kinematic viscosity of water = ANU

The computation stops if one of the following conditions arises

1. Wave steepness: $H/L = 0.172 \tanh kh$
2. Breaking depth: $db = 1.28\ Hb$
3. Orthogonal reaches the sides/shore/required depth.

The output of the model consists of grid locations of the orthogonals at different times, deformed wave crest direction, shoaling, refraction, friction, percolation and viscous dissipation coefficient and the resultant wave height.