ESTIMATION OF WAVE FORCES ON A TETHERED SPHERICAL FLOAT USING POTENTIAL FLOW THEORY

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ABSTRACT

Dynamic characteristics of offshore structures could be well studied, if wave forces are accurately predicted. As wave forces can be estimated more accurately from the pressure forces through potential flow theory than from Morison equation, potential flow theory is used to determine the wave forces acting on a tethered spherical float. Since the float size is much smaller than the wave length, diffraction effect is neglected. Wave forces are calculated based on the values of dynamic pressure and unit normal vector at various points considered on the cross sectional area of the float. Variations of wave forces with respect to wave height, wave period, water depth and float size are evaluated. Damping coefficients derived from the motion generated velocity potential are discussed and a relationship is established between wave force amplitude and damping coefficient.

INTRODUCTION

Wave forces arise due to the interaction of structure and waves, with the former modifying the kinematics and dynamics of the flow. In general, Morison equation is used for the computation of wave forces for small cylinders. Morison equation is based on the assumption that the incident wave field is not affected due to the presence of a structure. In the case of a tethered float, the vigorous oscillation of the float disturbes the incident wave field and hence potential flow theory is sought. The velocity potential defined in a wave field completely describes the kinematics and dynamics of flow within the fluid and its boundaries. In potential theory, it is assumed that the flow is inviscid, irrotational and incompressible so that the flow can be characterised by a velocity potential function, which satisfies the Laplace equation. Milgram (1976) and Garrison (1978) have discussed the validity of this assumption in calculating wave loads on offshore structures. Mathematical formulation of wave forces is relatively simple, if linear theory is used. The suitability and satisfactory accuracy in adopting linear wave theory for practical applications have been confirmed by the studies of Hogben et al (1977). In the present study, wave forces on a tethered spherical float, undergoing oscillatory motion in regular waves, have been determined using potential flow theory.

EARLIER WORK

Though wave length due to nonlinear wave theory is longer than that predicted by linear wave theory, the actual wave loading is underestimated by linear wave theory only by 5% (Hogben and Standing, 1975). Hence, linear wave theory, which is mathematically simple and well developed, has been followed in the analysis.

John (1950) has developed a Green function method to study the interaction of floating bodies with waves. Wave-structure interaction of a semi-immersed floating sphere which undergoes small heave oscillation has been discussed by Havelock (1955). Garrison (1974a) has presented numerical results of hydrodynamic forces exerted on a floating sphere. In the above investigations only freely floating body problems are considered as an approximation to slackly moored conditions. Jin (1976) has computed hydrodynamic forces for floating structures using potential flow theory. Garrison (1974b) developed a numerical procedure to estimate wave forces for arbitrary shapes of practical utility. Hogben et al (1977), Lighthill (1979) and Holmes et al (1983) have published comprehensive articles on hydrodynamic loading on offshore structures.

PROBLEM FORMULATION

A single tethered spherical float, undergoing oscillatory motion in regular waves, has been considered. The analysis is carried out in two-dimensional cartesian coordinate system with xy plane parallel to the horizontal plane of the sea surface. Basic assumptions involved in the analysis are:

(i) the tethered float moves in an inviscid and incompressible fluid.
(ii) the flow is irrotational.
(iii) water depth is uniform and finite.
(iv) incident waves are regular and unidirectional.
(v) amplitude of incident waves and body motions are small compared to wave length.
(vi) tether is assumed weightless.

Wave forces consist of two components: (i) force due to incident wave potential, and (ii) force due to diffracted wave potential. As the float size is much smaller than wave length diffraction effect is neglected. The velocity potential of small amplitude incoming wave propagating in the positive x-direction is given by,

$$\Phi_I(x,y,z,t) = Re\left\{\phi_I(x,y,z)\, e^{-j\omega t}\right\} \quad \ldots (1)$$

where,

$$\phi_I = \frac{gA}{\omega}\frac{\cosh k(z+d)}{\cosh kd} \quad \ldots (2)$$

g = acceleration due to gravity
A = wave amplitude
k = wave number $= 2\pi/\lambda$
λ = wave length
d = water depth
ω = radian frequency $= 2\pi/T$
T = wave period

For surface conditions (z=0),

$$\Phi_I(x,y,t) = Re\left\{\phi_I(x,y)\, e^{-j\omega t}\right\} \quad \ldots (3)$$

Hydrodynamic pressure due to incident wave potential is expressed as,

$$P_I(x,y,t) = Re\left\{P_i(x,y)\, e^{-j\omega t}\right\} \quad \ldots (4)$$

From the linearised form of Bernoulli's equation, the dynamic pressure is obtained as,

$$p_I(x,y,t) = -\rho \frac{\partial \Phi_I}{\partial t}$$

$$= -\rho \left[Re \frac{\partial}{\partial t} \left\{ \phi_I(x,y)\, e^{-j\omega t} \right\} \right]$$

$$\text{i.e } p_I(x,y,t) = j\omega\rho \ Re \left\{ \phi_I(x,y)\, e^{-j\omega t} \right. \qquad \text{... (5)}$$

Comparing equations (4) and (5),

$$p_I = j\omega\rho\, \phi_I \qquad \text{... (6)}$$

Wave forces (f_w) is computed from the pressure due to incident wave potential as,

$$f_w = \int_{C_o} p_I\, n_1\, ds \qquad \text{... (7)}$$

where,

n_1 = x component of unit normal vector

ds = incremental arc length on the body contour C_o

C_o = immersed portion of the cross sectional contour of the float

Pressure forces can be estimated accurately from potential flow theory than from Morison equation.

ESTIMATION OF WAVE FORCES

Wave forces acting on the float are estimated using eqn. (7). The cross section of the float is divided into number of segments as shown in Fig. 1. The boundary condition on the body surface is

$$\frac{\partial \phi_I}{\partial n} = n \qquad \text{... (8)}$$

where, n = unit normal vector

The unit normal vector can be resolved into two components along x and y axes as,

$$\frac{\partial \phi_I}{\partial x} = n_1 \qquad \frac{\partial \phi_I}{\partial y} = n_2$$

It is assumed that the waves are propagating in the positive x direction.
Therefore,

$$\frac{\partial \phi_I}{\partial y} = 0$$

From the geometry of Fig. 1,

$$\frac{\partial \phi_I}{\partial x} = n_1 = \sin\theta \qquad \text{... (9)}$$

where, θ is the angle between the x-axis and the tangent at a point on the body surface. i-1, i, i+1 ..... (Fig. 1) are the points considered on the cross sectional area of the float at regular intervals (=ds). Dynamic pressure acting at these points can be estimated using eqn. (6). Knowing the values of pressure, unit normal vector and incremental arc length, wave forces at these points can be computed. Sum of the forces over these points gives the total force acting on the float.

RESULTS AND DISCUSSION

Variation of wave forces has been studied for various wave periods (T), wave heights (H), sizes of the float (radius=R) and water depths (d). The results are shown in Figs. 2(a) to 2(d). The results show that with respect to wave period, wave forces initially increase till T = 7 s and thereafter no substantial increase is observed. Wave forces increase steadily with respect to wave height and size of the float. Influence of wave height on wave forces is quite large compared to wave period or float size. No variation in wave force is observed with change in water depth.

Non-dimensional wave forces are plotted against non-dimensional wave, structure and depth parameters (Figs. 3(a) to 3(d)). The terms are non-dimensionalised as follows:

Wave force, $\bar{f}_w = f_w / \rho g H^2 R$

Wave period, $\bar{T} = T/(\lambda/u)$

Wave height, $\bar{H} = H/gT^2$

Float radius, $\bar{R} = R/H$

Water depth, $\bar{d} = d/gT^2$

where, u = horizontal water particle velocity

Newman (1962) showed that there exists a relationship between the wave exciting force amplitude ($|f_w|$) and damping coefficient (β). For a two-dimensional problem, it can be written as,

$$|f_w| = A\sqrt{\frac{\rho g^2}{\omega}\beta} \qquad \text{... (10)}$$

Applying this relationship to the present case,

$$|f_w| = 3.5\,A\sqrt{\frac{\rho g^2}{\omega}\beta} \qquad \text{... (11)}$$

Damping coefficient is determined from the imaginary part of the motion generated velocity potential (ϕ_H) as described by Lee (1976),

$$\beta = \rho\omega \int_{C_0} \phi_H\, n_1\, ds \qquad \text{... (12)}$$

Method of estimating motion generated velocity potential is not given here as it is beyond the scope of the present paper. Wave forces ($|f_w'|$) estimated from the motion generated velocity potential using eqn. (12) is presented along with the wave forces computed from the incident wave potential ($|f_{w_i}|$) in Table 1. Both results agree well and confirm the accuracy of potential flow approach.

Very few studies have been reported on fluid-structure interaction on spherical shaped bodies, probably due to their rare utility in offshore activities. However, there are spherical shaped structures such as data buoys, marine observation buoys, tethered float breakwaters, etc. for which a realistic analysis is very much essential. The study presented in this paper will be useful to estimate wave forces and to analyse response of the structures to ocean waves.

CONCLUSIONS

Wave forces on a tethered spherical float have been determined using potential flow theory for various wave periods, wave heights, float sizes and water depths. A relationship is established between wave force amplitude and damping coefficient which shows that wave forces can also be indirectly estimated from the motion generated velocity potential. This method can also be applied to estimate wave forces on large spherical shaped bodies, incorporating diffracted wave potential.

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Table 1 : Comparison of wave forces calculated from incident and motion generated wave potentials

T(s)	β	$\|f_w\|$	$\|f'_w\|$
4	1691.15	15924.98	17846.90
5	1651.97	18200.44	19550.30
6	1456.28	19559.62	20086.02
7	1274.34	20414.39	20090.80
8	1125.02	20981.34	20091.10
9	1004.30	21374.71	20091.10
10	905.87	21658.09	20091.10

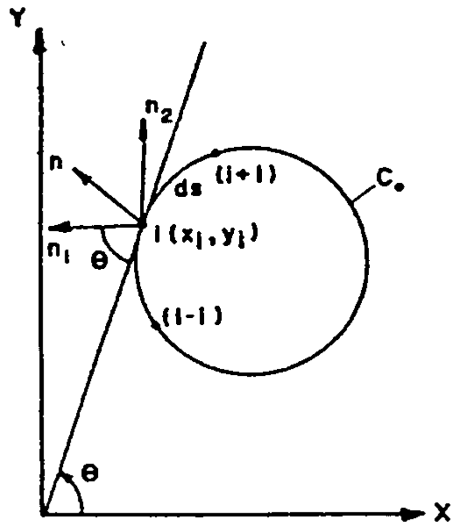

Fig. 1 Definition sketch for wave force computation

θ = angle between the x-axis and the tangent at the point (x_i, y_i)

n = unit normal vector at the point (x_i, y_i)

ds = arc length between two points.

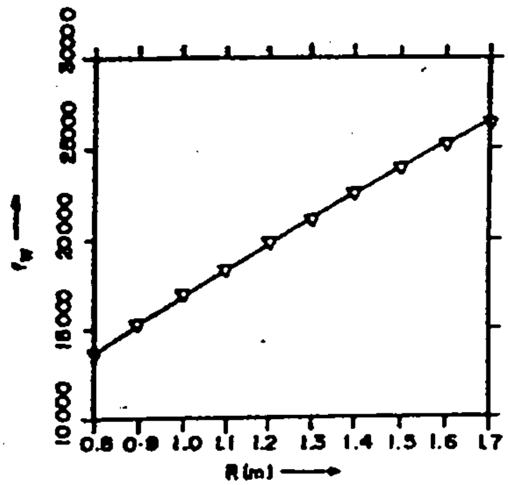

Fig. 2c Variation of wave force with float size

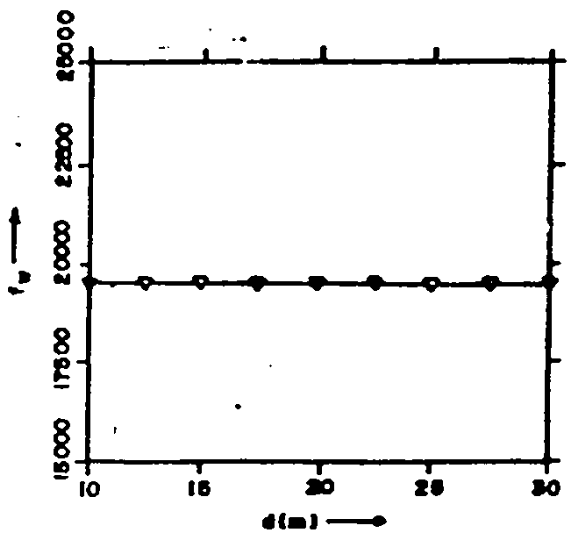

Fig. 2d Variation of wave force with water depth

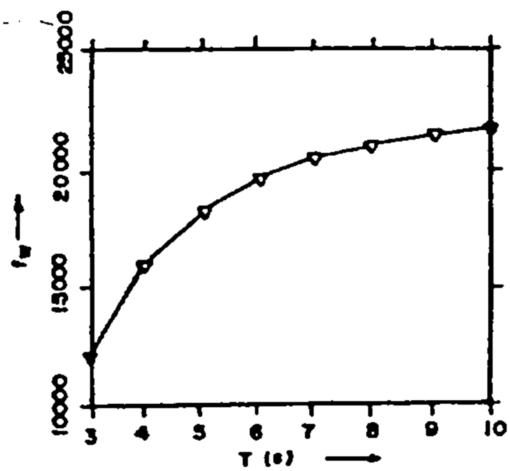

Fig. 2a Variation of wave force with wave period

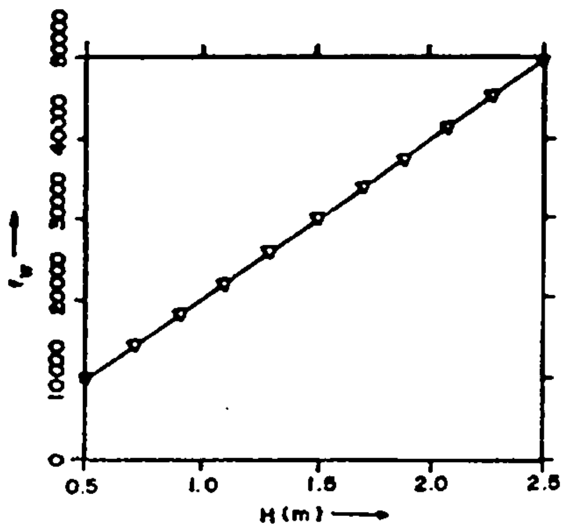

Fig. 2b Variation of wave force with wave height

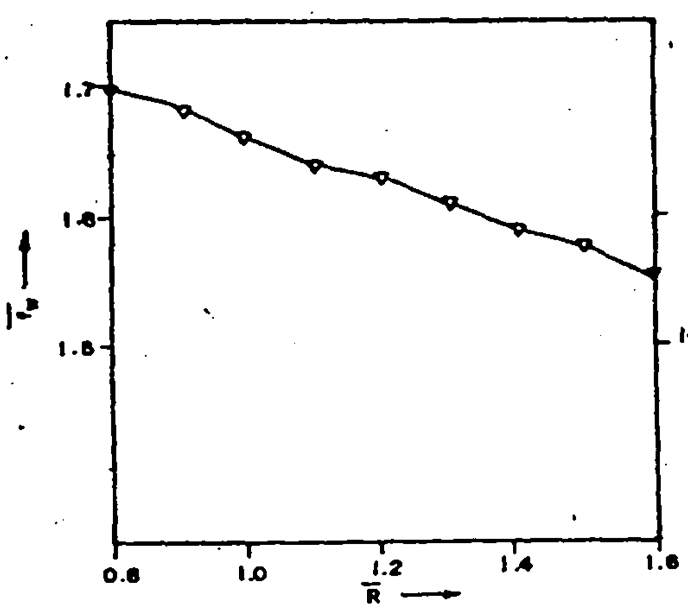

Fig. 3c Variation of wave force (f_w) with float size (R) (non-dimensionalised)

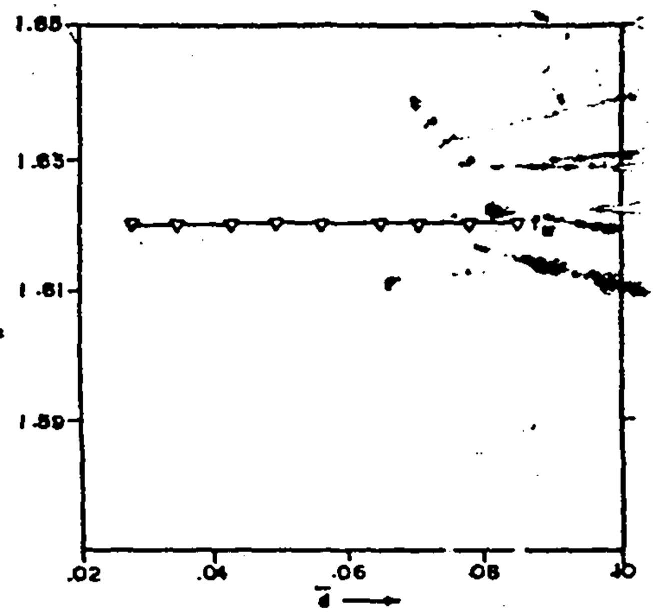

Fig. 3d Variation of wave force (f_w) with water depth (d) (non-dimensionalised)

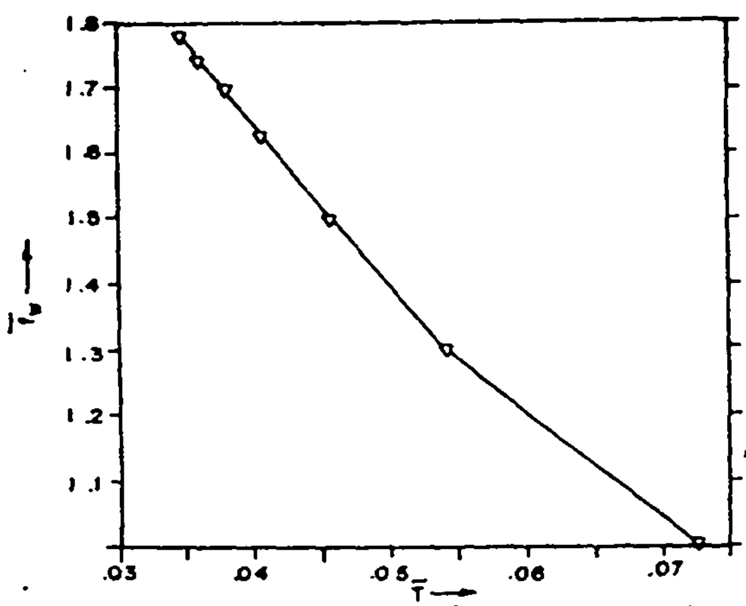

Fig. 3a Variation of wave force (f_w) with wave period (T) (non-dimensionalised)

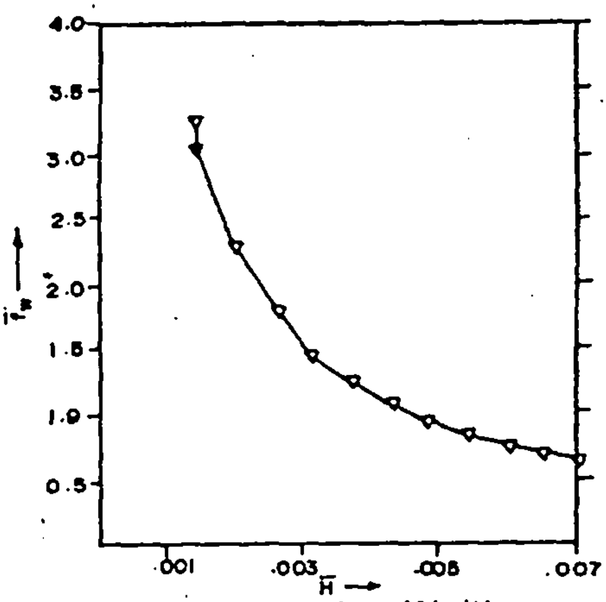

Fig. 3b Variation of wave force (f_w) with wave height (H) (non-dimensionalised)